LDQCM-15 Workshop
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TECHNISCHE UNIVERSITÄT DRESDEN

Topology by Dissipation in Atomic Fermion Systems: Dissipative Chern Insulators

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Collaborations:
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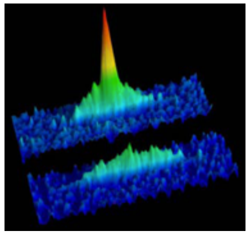
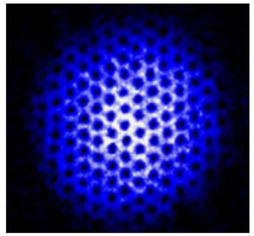
FWF Der Wissenschaftsfonds.

Motivation

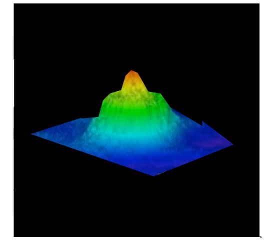


Bose-Einstein Condensate (1995)

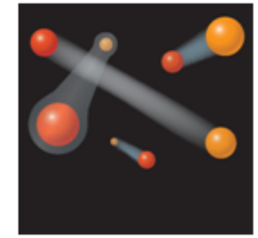


Vortices (1999)

Many-body physics with cold atoms



Mott Insulator (2002)



Fermion superfluid (2003)

Common theme:

many-body system

Temperature T, particle number N

- closed system (isolated from environment)
- stationary states in thermodynamic equilibrium

➡ thermalization/equilibration (PennState, Berkeley, Chicago, ...)

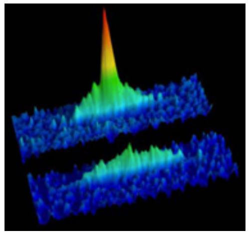
➡ sweep and quench many-body dynamics (Munich, Vienna)

➡ metastable excited many-body states (Innsbruck, MIT, ...)

➡ ...

Motivation

Many-body physics with cold atoms



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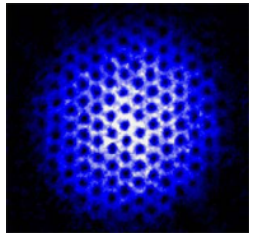

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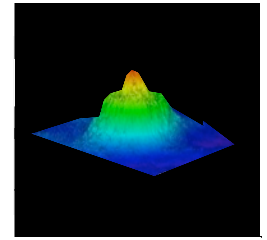

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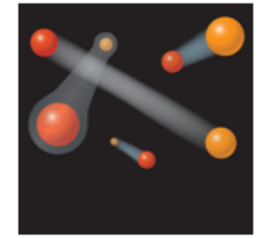

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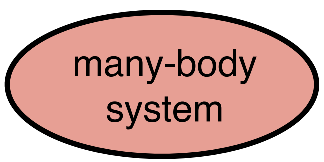
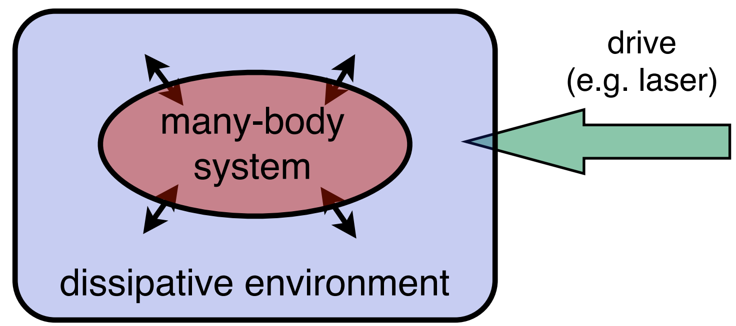


Temperature T, particle number N

- closed system (isolated from environment)
- stationary states in thermodynamic equilibrium

Novel Situation: Cold atoms as open many-body systems



- natural occurrences of dissipation

➡ no immediate condensed matter counterpart

- use manipulation tools of quantum optics

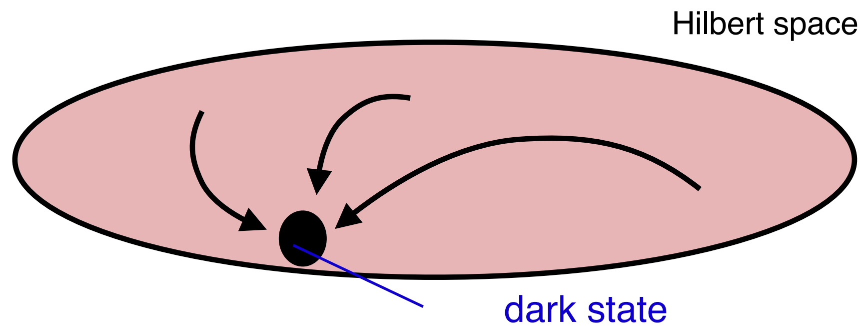
➡ drive/dissipation as dominant resource of many-body dynamics!

Motivation: Topology by Dissipation

Basic Setting: Thinning out a density matrix to a pure state (“cooling”)

$$\rho \xrightarrow{t\to\infty} |D\rangle\langle D|$$

mixed pure



bosonic systems
SD et al. Nat. Phys. (2008)
F. Verstraete et al. Nat. Phys. (2009)

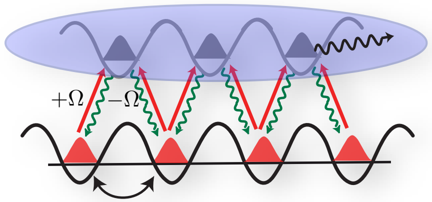
spin systems
Krauter et al. PRL (20010)
Barreiro et al. Nature (2011)

fermion systems
SD et al. PRL (2010)
SD et al. Nat. Phys. (2011)

Key Questions:

- Is topological order an exclusive feature of Hamiltonian ground states, or pure states?
  related: Kitagawa, Berg, Rudner, Demler, PRB (2010); Lindner, Refael, Galitski, Nature Phys. (2011).
  Kapit, Hafezi, Simon, PRX (2014).
- Which topological states be reached by a targeted, dissipative cooling process?
- What are proper microscopic, experimentally realizable models?
- What are the parallels and differences to the equilibrium (ground state) scenario?

Outline

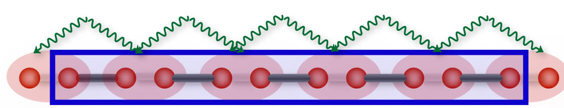


Order by dissipation

↓

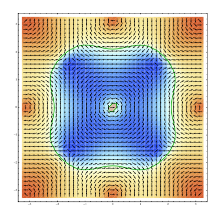
Topology by dissipation

One Dimension



SD, E. Rico, M. A. Baranov, P. Zoller, Nat. Phys. (2011)

Two Dimensions



C. Bardyn, E. Rico, M. Baranov, A. Imamoglu, P. Zoller, SD, PRL (2012); New J. Phys. (2013);
J. C. Budich, P. Zoller, SD, PRA (2015)

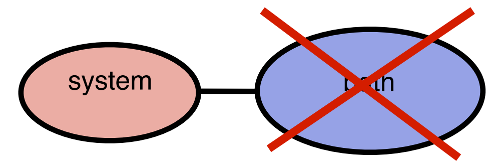
Dissipative Chern insulators

Many-Body Physics with Dissipation: Description

- Many-body master equations

Lindblad operators

$$\partial_t \rho = \underbrace{-i[H,\rho]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})}_{\text{dissipative evolution}}$$



$\mathcal{L}[\rho]$ -- Liouvillian operator

➡ extend notion of Hamiltonian engineering to dissipative sector

➡ microscopically well controlled non-equilibrium many-body quantum systems

➡ here: focus on H = 0

- Important concept: Dark states

$$L_i |D\rangle = 0 \quad \forall i$$

$$\Rightarrow \mathcal{L}[|D\rangle\langle D|] = 0$$

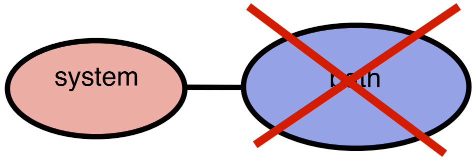
➡ time evolution stops when $\rho = |D\rangle\langle D|$

Many-Body Physics with Dissipation: Description

- Many-Body master equations

$$\partial_t \rho = -i[H,\rho] + \kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})$$

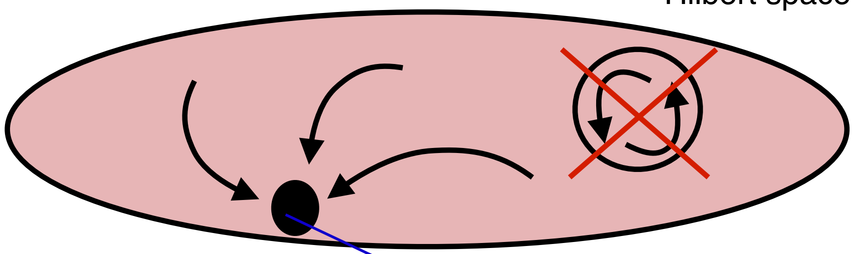
Lindblad operators



$\mathcal{L}[\rho]$ -- Liouvillian operator

- Interesting situation: unique dark state solution

B. Kraus, SD et al. PRA 08



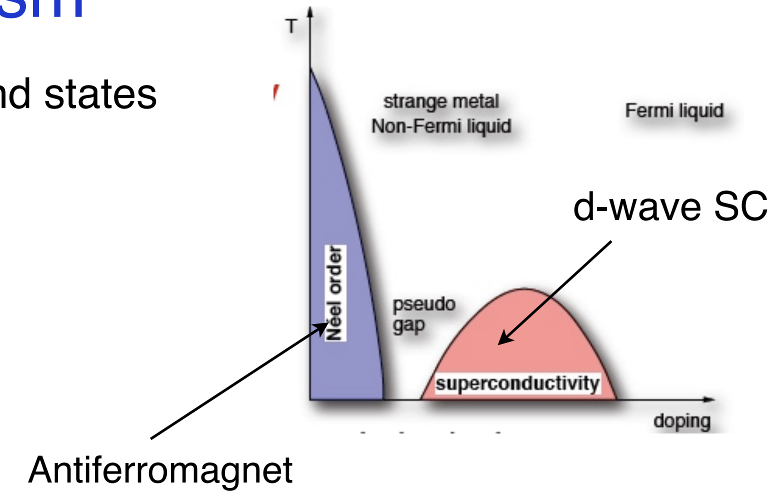
- dark subspace one-dimensional
- no other stationary solutions

➡ directed motion in Hilbert space $\rho \stackrel{t\to\infty}{\longrightarrow} |D\rangle\langle D|$

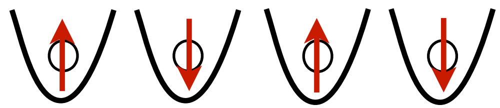
➡ dissipation increases purity (entropy pump)

SD, W. Yi, A. Daley, P. Zoller, PRL 105 (2010)

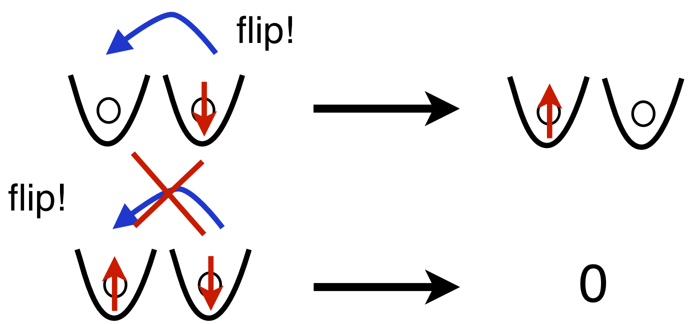
Paired Fermionic Dark States: Mechanism



- proximity of magnetic and superconducting order in fermion ground states
- Antiferromagnetic Neel state (half filling)



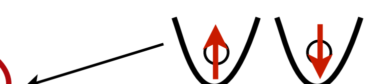
➡ Lindblad operators: $\ell_{i-}^{+} = c_{i-1,\uparrow}^{\dagger} c_{i,\downarrow}$



➡ magnetic dark state based on Fermi statistics

- Superconducting state: delocalized Neel order

$$|\mathrm{BCS}_1\rangle = (d^\dagger)^N |\mathrm{vac}\rangle, \;\; d^\dagger = \sum_i (c_{i+1,\uparrow}^\dagger + c_{i-1,\uparrow}^\dagger) c_{i,\downarrow}^\dagger$$



➡ Lindblad operators: $L_i^+ = \ell_{i,+}^+ + \ell_{i,-}^+ = (c_{i+1,\uparrow}^\dagger + c_{i-1,\uparrow}^\dagger) c_{i,\downarrow}$

➡ sc dark state based on additional phase locking

➡ Combine fermionic Pauli blocking with phase locking

Dissipative Pairing: Set of Lindblad Operators

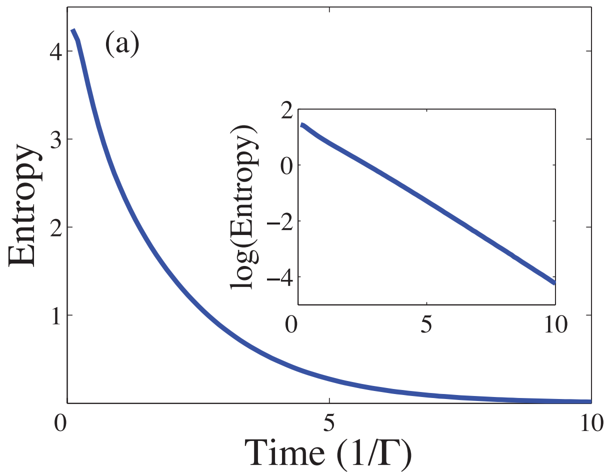
- The full set of Lindblad operators is found from

$$[L_i^\alpha, G^\dagger] = 0 \quad \forall i, \alpha \qquad\qquad |D(N)\rangle \sim G^{\dagger\,N}|\mathrm{vac}\rangle$$

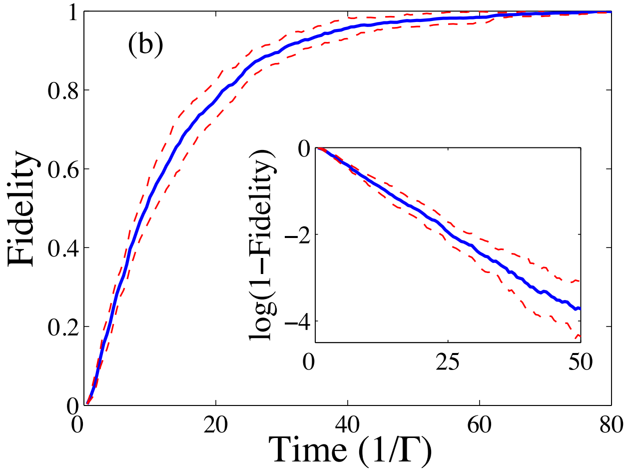
- given by

Pauli matrices

$$\boxed{L_i^\alpha = (c_{i+1}^\dagger + c_{i-1}^\dagger)\sigma^\alpha c_i} \qquad c_i = \begin{pmatrix} c_{\uparrow,i} \\ c_{\downarrow,i} \end{pmatrix}$$



entropy pump



unique dark state

➡ Projective pair condensation mechanism, does not rely on attractive conservative forces

Fixed Number vs. Fixed Phase Lindblad Operators

- spinless fermions for simplicity
- fixed number Lindblad operators

$$L_i = C_i^\dagger A_i$$

- resulting dark state

$$|BCS, N\rangle = G^{\dagger\, N}|\text{vac}\rangle$$

$\Leftrightarrow$

- fixed phase Lindblad operators

$$\ell_i = C_i^\dagger + re^{\mathrm{i}\theta} A_i$$

- resulting dark state (with $\Delta N \sim 1/\sqrt{N}$

$$|BCS, \theta\rangle = \exp(re^{\mathrm{i}\theta} G^\dagger)|\text{vac}\rangle$$

- requirements

translation invariant creation and annihilation part

$$C_i^\dagger = \sum_j v_{i-j} a_j^\dagger \qquad C_k^\dagger = v_k a_k^\dagger$$

$$A_i = \sum_j u_{i-j} a_j \qquad A_k = u_k a_k$$

antisymmetry

$$\varphi_k = \frac{v_k}{u_k} = -\varphi_{-k}$$

$$G^\dagger = \sum_k \varphi_k c_{-k}^\dagger c_k^\dagger$$

- comment: construct exactly solvable interacting Hubbard models with parent Hamiltonian

exact number conserving Majorana wavefunction: Iemini, Mazza, Rossini, SD, Fazio, arxiv (2015)

$$H = \sum_i L_i^\dagger L_i \qquad L_i|D\rangle = 0\,\forall i$$

Spontaneous Symmetry Breaking and Dissipative Gap

- use equivalence of fixed number and fixed phase states in thdyn limit
- use exact knowledge of stationary state: linearized long time evolution

$$\mathcal{L}[\rho] = \kappa \sum_i [\ell_i \rho \ell_i^\dagger - \tfrac{1}{2}\{\ell_i^\dagger \ell_i, \rho\}] = \sum_{\mathbf{q}} \kappa_{\mathbf{q}} [\ell_{\mathbf{q}} \rho \ell_{\mathbf{q}}^\dagger - \tfrac{1}{2}\{\ell_{\mathbf{q}}^\dagger \ell_{\mathbf{q}}, \rho\}]$$

- properties
  - relation to microscopic operators

fixed by average particle number

fixed spontaneously

$$L_i = C_i^\dagger A_i \xrightarrow{t \to \infty} \ell_i = C_i^\dagger + r e^{i\theta} A_i$$

"low energy limit"

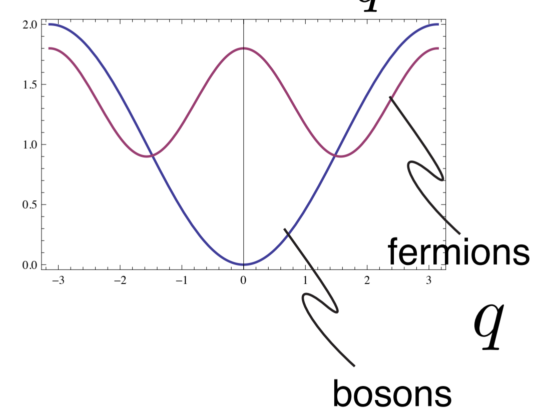
  - effective fermionic quasiparticle operators

$\ell_{\mathbf{q}} |BCS, \theta\rangle = 0$; fulfill Dirac algebra -> uniqueness

  - dissipative gap in the damping rate

$$\kappa_{\mathbf{q}} = \kappa_0 \int_{\mathrm{BZ}} \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{|u_{\mathbf{k}} v_{\mathbf{k}}|^2}{|u_{\mathbf{k}}|^2 + |\alpha v_{\mathbf{k}}|^2} (|u_{\mathbf{q}}^2| + |v_{\mathbf{q}}^2|) \geq \kappa_0 n$$

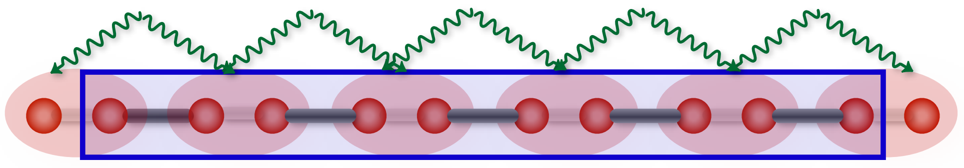
damping rate κ_q



➡ Scale generated in long time evolution ; exponentially fast approach of steady state

➡ Robustness of prepared state against perturbations

Topology by Dissipation: Dissipative Kitaev Wire



SD, E. Rico, M. A. Baranov, P. Zoller, Nat. Phys. 7, 971 (2011)

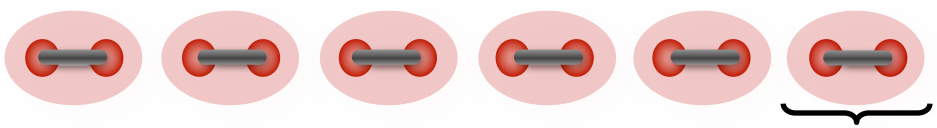
Kitaev's quantum wire (Hamiltonian scenario)

- spinless superconducting fermions on a lattice   Kitaev (2001)
- Hamiltonian in Bogoliubov basis $H \sim \sum(\tilde{a}_i^\dagger \tilde{a}_i - \frac{1}{2})$ $\tilde{a}_i|G\rangle = 0\,\forall\, i$
- two inequivalent representatives

quasilocal!

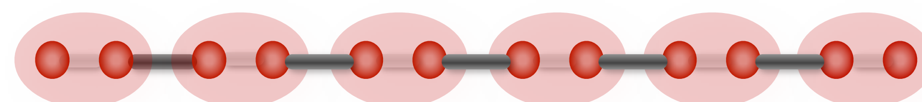
$\tilde{a}_i = a_i$

$\tilde{a}_i = \frac{1}{2}(a_{i+1} + a_{i+1}^\dagger - a_i + a_i^\dagger)$



physical site

trivial phase



nontrivial phase

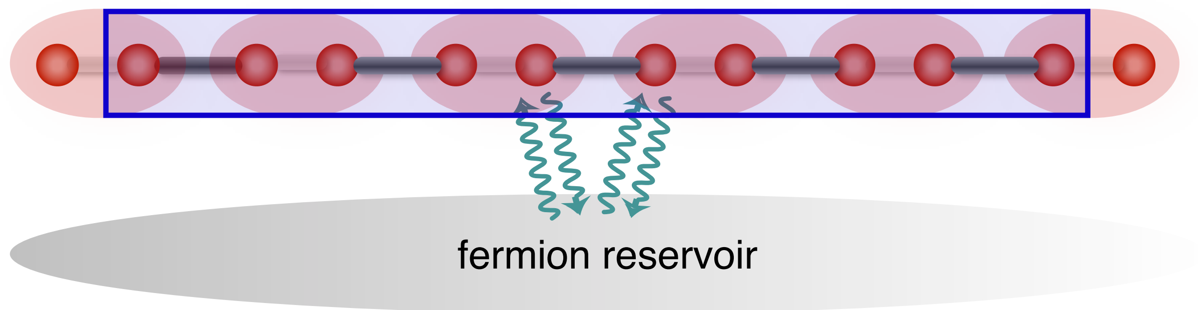
bulk

- p-wave superfluid in ground state
- gapped spectrum

edge

- unpaired zero energy Majorana edge modes, or
- non-local Bogoliubov fermion

Dissipative Majorana Quantum Wire



- Kitaev's Bogoliubov operators as Lindblad operators $\tilde{a}_i = \frac{1}{2}(a_{i+1} + a^\dagger_{i+1} - a_i + a^\dagger_i)$ quasilocal

$$L_i = \tilde{a}_i$$

- master equation

$$\dot{\rho} = \kappa \sum_{i=1}^{N-1} \left(\tilde{a}_i \rho \tilde{a}_i^\dagger - \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \rho - \rho \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \right)$$

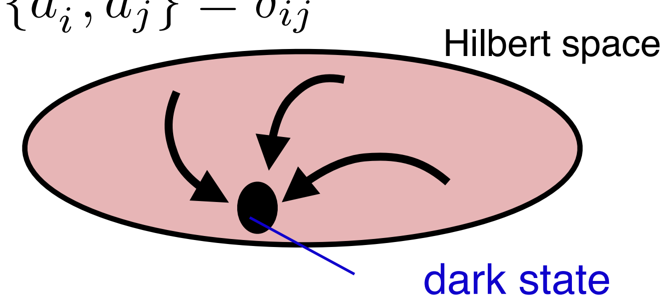
bulk driven to pure steady state:
Kitaev's ground state

$$\tilde{a}_i |\mathrm{p-wave}\rangle = 0 \ (i = 1, \ldots, N-1)$$

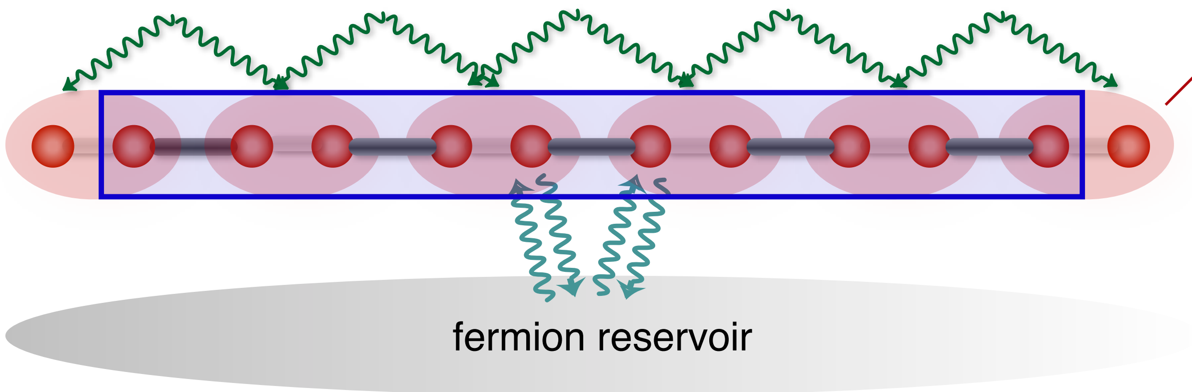
dark state = topological p-wave

$\{\tilde{a}_i, \tilde{a}_j\} = 0 \quad \{\tilde{a}_i^\dagger, \tilde{a}_j\} = \delta_{ij}$

=> dark state unique



Dissipative Majorana Quantum Wire



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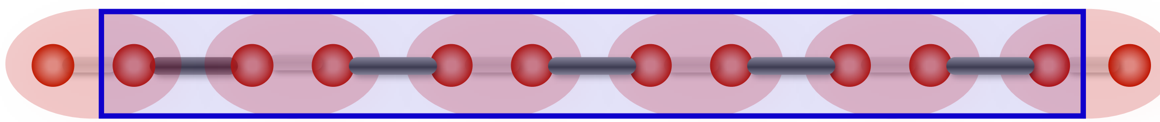
dark state = topological p-wave

Majorana edge modes decoupled from dissipation

$$|0\rangle, \ |1\rangle = \tilde{a}_N^\dagger |0\rangle$$

non-local decoherence free subspace

Dissipative Majorana Quantum Wire



Edge - Bulk:

- dynamically isolated from each other

$$\rho_{\text{bulk-edge}} \lesssim e^{-\lambda_{\text{gap}} t} \rho_{\text{bulk-edge}}(0) \to 0$$

$$\Rightarrow t \to \infty : \rho \to \rho_{\text{edge}} \otimes \rho_{\text{bulk}}$$

- edge mode subspace protected by dissipative gap

➡ parallels Hamiltonian case

✓ robustness against disorder/ mixed states

✓ non-abelian exchange statistics

✓ topological invariant

$$\rho_{\text{bulk}}(\infty) = |\text{p}-\text{wave}\rangle\langle \text{p}-\text{wave}|$$

bulk cooled to pure steady state: Kitaev's ground state

$$\tilde{a}_i |\text{p}-\text{wave}\rangle = 0 \ (i = 1, \ldots, N-1)$$

dark state = topological p-wave

$$\dot{\rho}_{\text{edge}} = 0 \quad (\rho_{\text{edge}})_{\alpha\beta} \equiv \langle \alpha | \rho_{\text{edge}} | \beta \rangle \quad |\alpha\rangle \in \{|0\rangle, |1\rangle\}$$

Majorana edge modes decoupled from dissipation

$$|0\rangle,\ |1\rangle = \tilde{a}_N^\dagger |0\rangle$$

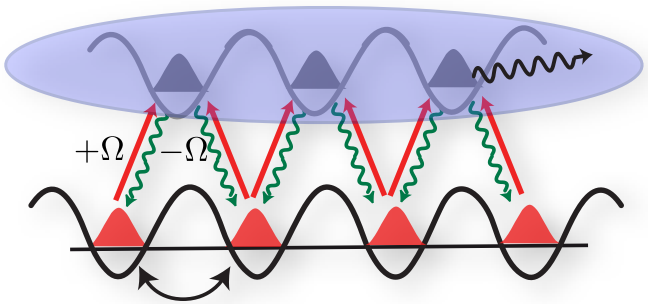
non-local decoherence free subspace

Implementation with Fermionic Atoms

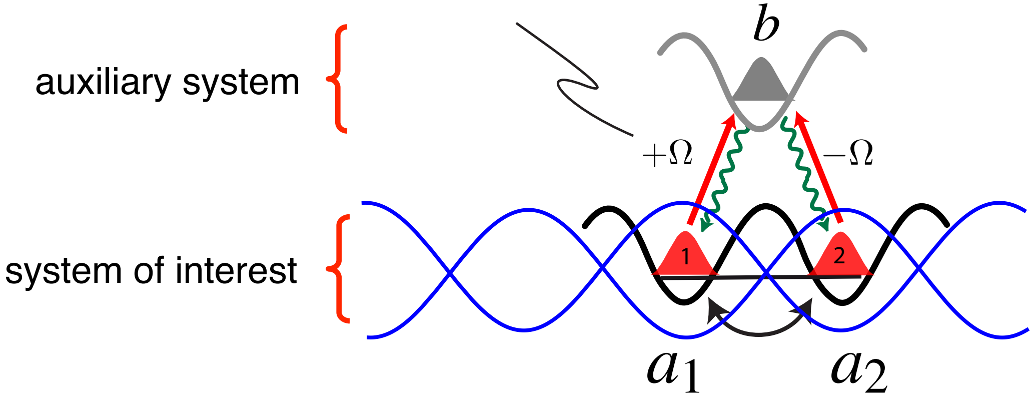
- We propose microscopically

$$J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1})$$

by immersion of driven system into BEC reservoir

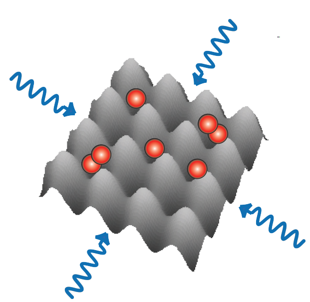


(i) Drive: coherent coupling to auxiliary system with double wavelength Raman laser



$$\lambda_{\text{laser}} = 2\lambda_{\text{lattice}}$$

driving laser



Implementation with Fermionic Atoms

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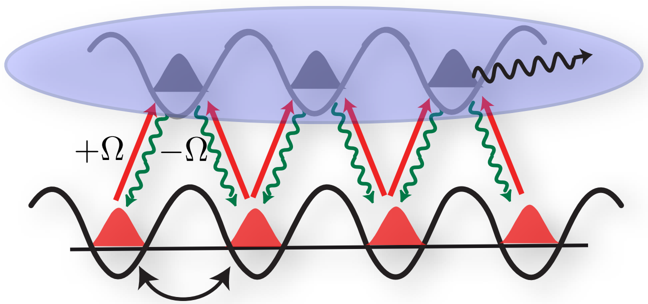
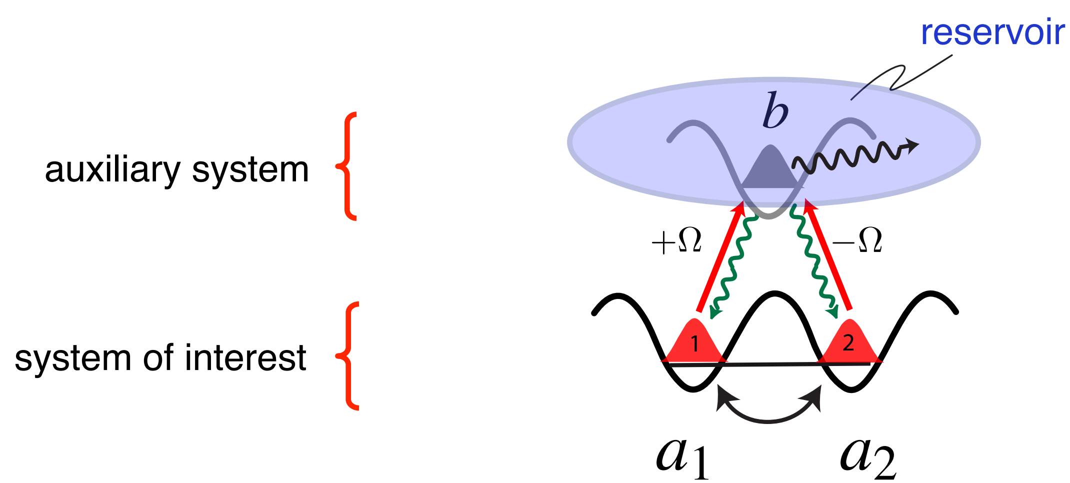
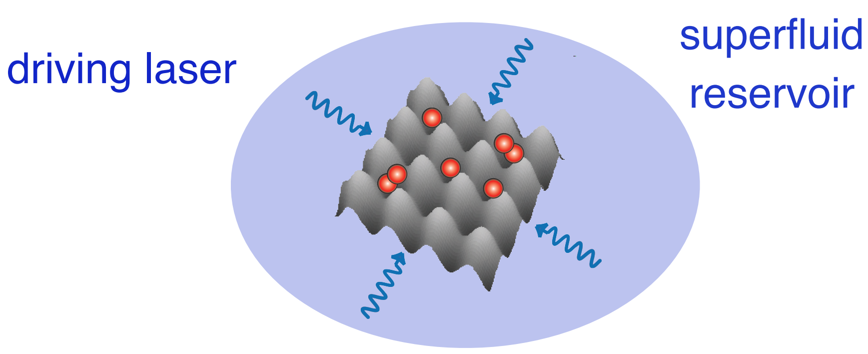


(ii) Dissipation: phonon emission into superfluid reservoir





Implementation with Fermionic Atoms

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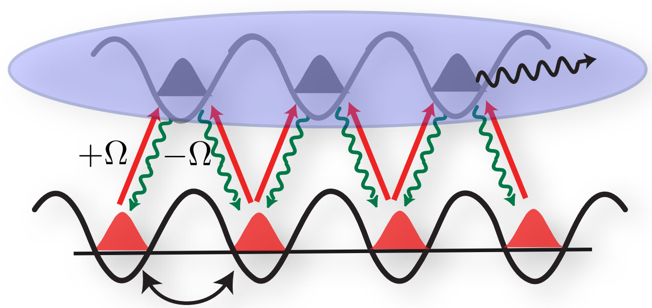


- Connection to quadratic theory: we obtain

fixed number

fixed phase

long times

$$J_i = \underbrace{(a_i^\dagger + a_{i+1}^\dagger)}\underbrace{(a_i - a_{i+1})} \longrightarrow j_i = (\underbrace{a_i^\dagger + a_{i+1}^\dagger} + \underbrace{a_i - a_{i+1}})$$

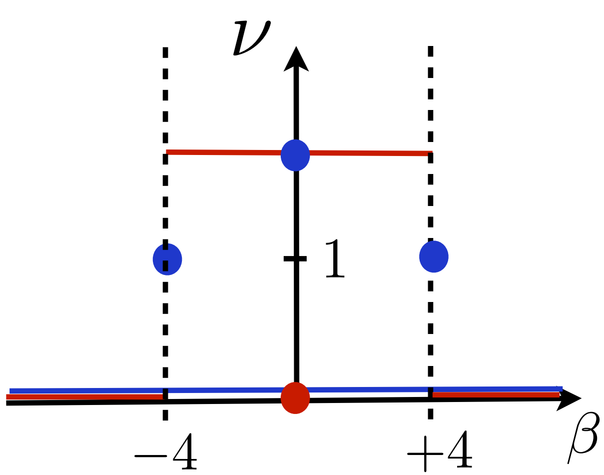
$\hat{=}$ “low energies”

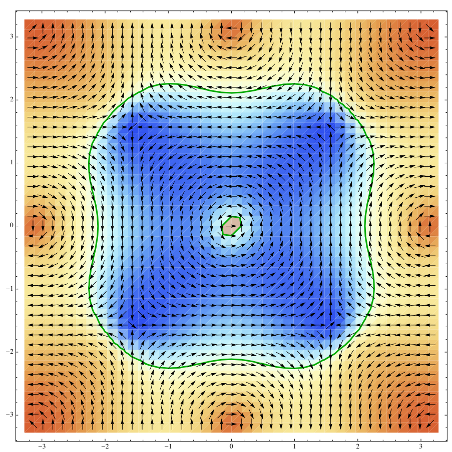
$\propto \tilde{a}_i$

dissipative gap
emerges naturally

Kitaev’s Majorana operators

Topology by Dissipation:
Dissipative Chern Insulators





J. C. Budich, P. Zoller, SD, PRA (2015)

Dissipative Chern Insulators (BdG Superfluids/-conductors)

- Q: How general is the concept of “Topology by Dissipation”?

- recipe for pure dissipative topological states (so far)
  - Bogoliubov eigenoperators as Lindblad operators $H_{\text{parent}} = \sum_i L_i^\dagger L_i \quad L_i|G\rangle = 0 \forall i$
  - Hamiltonian ground state as dissipative dark state $|D\rangle = |G\rangle$
  - quasi-locality of Wannier functions key requirement for physical realization

$$L_i = \sum_j u_{j-i} a_j + v_{j-i} a_j^\dagger$$

- fundamental caveat:
  - no exponentially localized Wannier functions exist for states with non-vanishing Chern number
    - Landau levels: Wannier functions decay $\sim r^{-2}$ D. J. Thouless, J. Phys. C (1984);
    - general band structures C. Brouder et al. PRL (2007)

➡ topology interferes with natural locality of the Lindblad operators

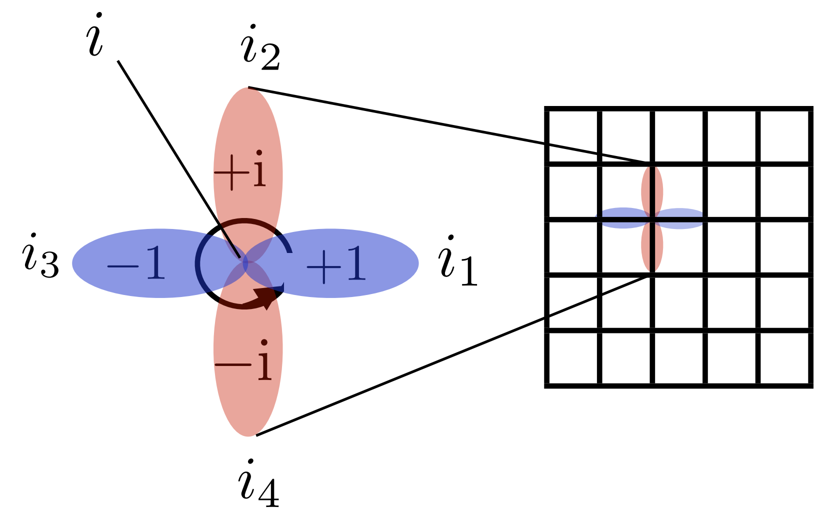
Model

- Strategy: combine
  - critical (topological) quasi-local Lindblad operators
  - non-topological Lindblad stabilizing critical point

- Lindblad operators generating dissipative dynamics:
  - starting point: interacting Liouvillian with $L_i = C_i^\dagger A_i$ & long time linearization
  - e.g. half filling $L_i = C_i^\dagger + A_i$
  - creation part

$$C_i^\dagger = \beta\, a_i^\dagger + (a_{i_1}^\dagger + a_{i_2}^\dagger + a_{i_3}^\dagger + a_{i_4}^\dagger)$$

s-wave symmetric creation part



  - annihilation part

$$A_i = (a_{i_1} + \mathrm{i} a_{i_2} - a_{i_3} - \mathrm{i} a_{i_4})$$ local circulation

$$= \nabla_{i,x} a_i + \mathrm{i}\nabla_{i,y} a_i$$ p-wave symmetric annihilation part

Observations

- pure stationary state: $\{L_i, L_j\} = 0, \ \{L_i, L_j^\dagger\} \neq 0 \ \forall i, j$
- standard 2D diagnostics via first Chern number

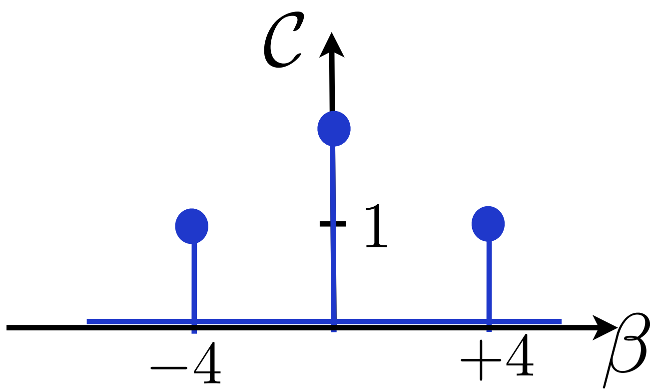
$$\mathcal{C} = \frac{1}{4\pi} \int d^2k \, \vec{n}_{\mathbf{k}} (\partial_{k_1} \vec{n}_{\mathbf{k}} \times \partial_{k_2} \vec{n}_{\mathbf{k}})$$

- $\vec{n}_{\mathbf{k}}$ characterizes the pure Gaussian state

$$\begin{pmatrix} \langle [a^\dagger_{\mathbf{k}}, a_{\mathbf{k}}] \rangle & \langle [a^\dagger_{\mathbf{k}}, a^\dagger_{-\mathbf{k}}] \rangle \\ \langle [a_{-\mathbf{k}}, a_{\mathbf{k}}] \rangle & \langle [a_{-\mathbf{k}}, a^\dagger_{-\mathbf{k}}] \rangle \end{pmatrix} = \vec{n}_{\mathbf{k}} \vec{\sigma}$$

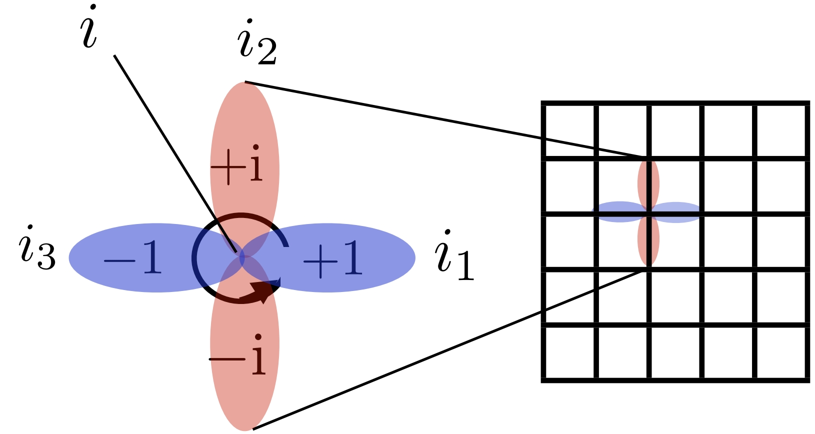
$|\vec{n}_{\mathbf{k}}| = 1$ pure state

- Chern number vanishes except for special points
- special points are critical: closing of damping gap



➡ but: Lindblad operators local, how can C be nonzero?

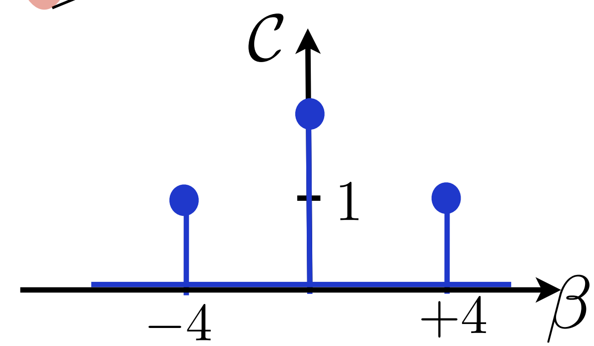
Physics at the dissipative critical point



- momentum space Lindblad operators

$$L_{\mathbf{k}} = \tilde{u}_{\mathbf{k}} a_{\mathbf{k}} + \tilde{v}_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}}$$

$$B_{\mathbf{k}} = \begin{pmatrix} \tilde{u}_{\mathbf{k}} \\ \tilde{v}_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} 2i\,(\sin(k_x) + i\sin(k_y)) \\ \beta + 2(\cos(k_x) + \cos(k_y)) \end{pmatrix}$$



- critical point $\beta = -4$: there is one point $\mathbf{k}_* = 0$ where

$$L_{\mathbf{k}_*} = 0$$

- overcompleteness of quasi-local (pseudo) Bloch/Wannier functions necessary to obtain nonzero Chern number

E. Rashba, L. Zhukov, A. Efros, PRB (1997)

- implies damping gap closing: $\kappa_{\mathbf{k}_*} = \{L^{\dagger}_{\mathbf{k}_*}, L_{\mathbf{k}_*}\} = 0$

➡ quasilocal Lindblad operators can support critical Chern states only

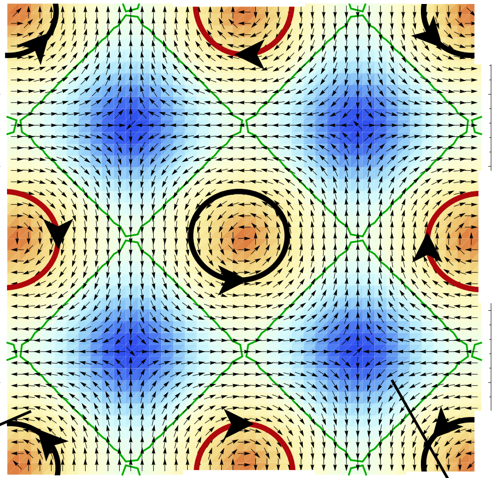
Stabilization of the critical point

- Chern number decomposition: sum of winding numbers around TRI points λ within "electron region" $\mathcal{E}$, where $\hat{n}_{3,\mathbf{k}} > 0$

$$\mathcal{C} = \frac{1}{4\pi}\int d^2\mathbf{k}\,\hat{\vec{n}}_{\mathbf{k}}(\partial_{k_1}\hat{\vec{n}}_{\mathbf{k}} \times \partial_{k_2}\hat{\vec{n}}_{\mathbf{k}}) = \sum_{\lambda\in\mathcal{E}} \nu_\lambda$$

$$\hat{\vec{n}}_k = \frac{\vec{n}_k}{|\vec{n}_k|}$$

$$\nu_\lambda = \frac{1}{2\pi}\oint_{\mathcal{F}_\lambda} \nabla_{\mathbf{k}}\theta_{\mathbf{k}} \cdot d\mathbf{k}$$



$n_{3,\mathbf{k}} < 0$

non-critical

$\mathcal{C} = 0$

$\mathcal{E}$

$\hat{n}_{3,\mathbf{k}} > 0$

vector field: $\begin{pmatrix} n_{1,\mathbf{k}} \\ n_{2,\mathbf{k}} \end{pmatrix} = r_{\mathbf{k}} \begin{pmatrix} \sin\theta_{\mathbf{k}} \\ \cos\theta_{\mathbf{k}} \end{pmatrix}$

height function: $\hat{n}_{3,\mathbf{k}} = 1 - 2n(\mathbf{k})$ — fermion occ.

Stabilization of the critical point

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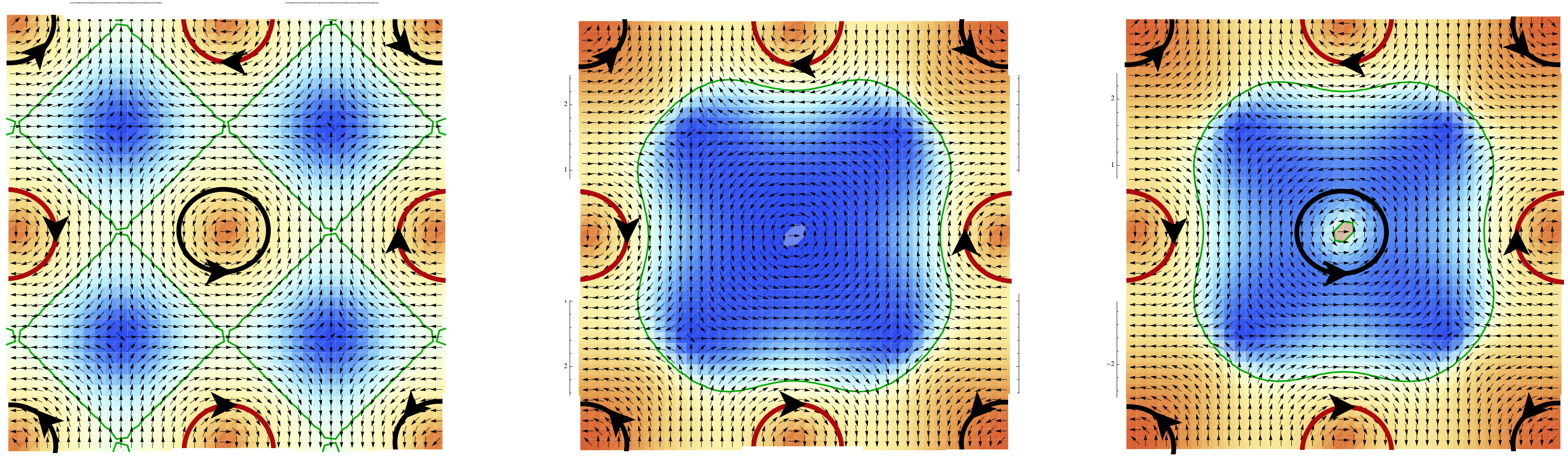


non-critical $\mathcal{C} = 0$ | critical $\mathcal{C} = -1$ | near critical $\mathcal{C} = 0$

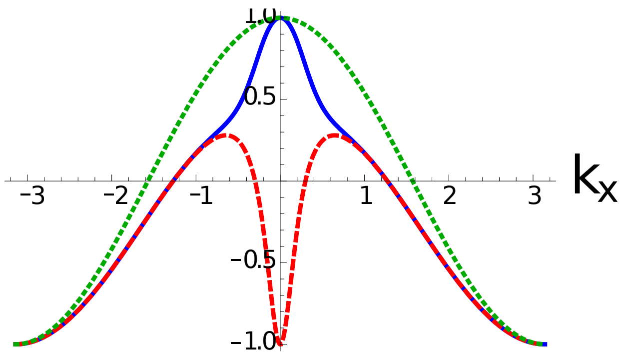
➡ need to "plug the hole" (here, near k=0)

Dissipative Hole Plugging

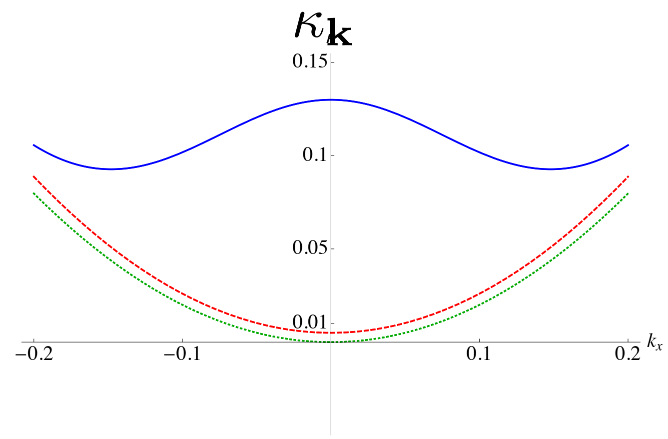
- minimal solution: add momentum selectively non-topological Lindblad operators

$$L_{\mathbf{k}}^{A} = \sqrt{g} e^{-\mathbf{k}^2 d^2} a_{\mathbf{k}}$$

- result: $n_{3,\mathbf{k}} = 1 - 2n_{\mathbf{k}}$

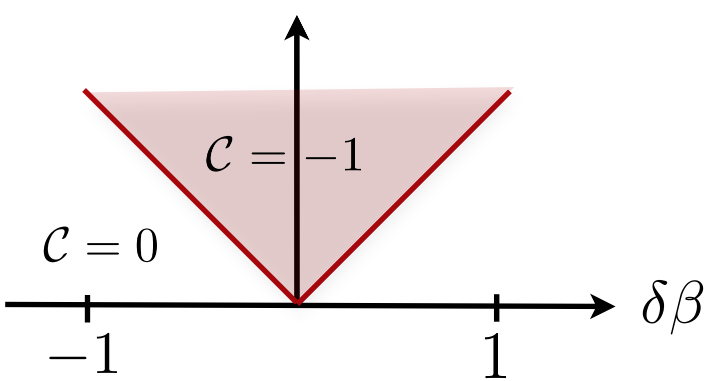


hole plugging



finite damping gap

- phase diagram



deviation from critical point

➡ dissipative stabilization of a critical topological point into a phase

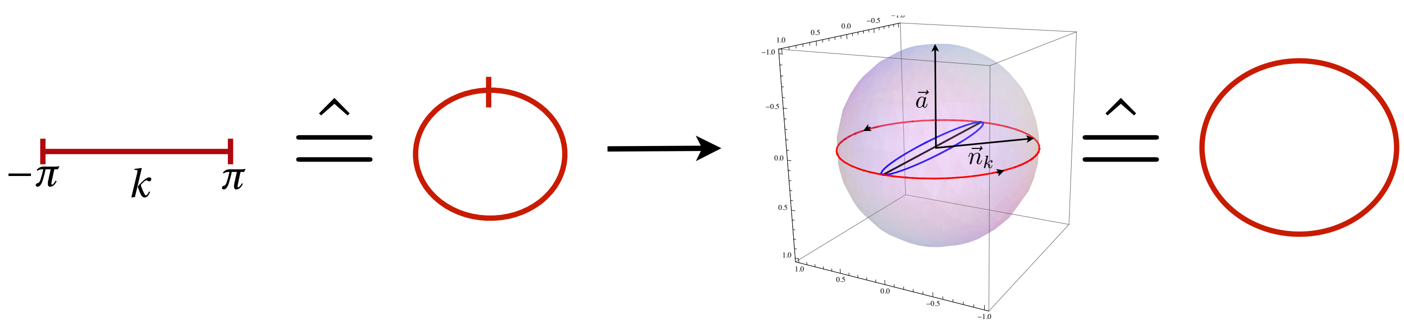
Nature of the Dissipative Topological Phase Transition

- Topological stability requires additional “purity gap” for mixed density matrix

- A Gaussian translationally invariant state is completely characterized by:

$$\begin{pmatrix} \langle [a_k^\dagger, a_k] \rangle & \langle [a_k^\dagger, a_{-k}^\dagger] \rangle \\ \langle [a_{-k}, a_k] \rangle & \langle [a_{-k}, a_{-k}^\dagger] \rangle \end{pmatrix} = \vec{n}_k \vec{\sigma} = Q_k \qquad |\vec{n}_k| \leq 1 \quad \forall k \in (-\pi, \pi]$$

- mapping circle to circle (chiral symmetry) $\vec{n}_k : S^1 \to S^1$ (pure states, $|\vec{n}_k| = 1$)



- Winding number topological invariant

$$W = \frac{1}{4\pi \mathrm{i}} \int_{-\pi}^{\pi} dk \, \mathrm{tr}(\Sigma Q_k^{-1} \partial_k Q_k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \vec{a} \cdot \left(\hat{\vec{n}}_k \times \partial_k \hat{\vec{n}}_k \right)$$

Topological invariant for mixed density matrices

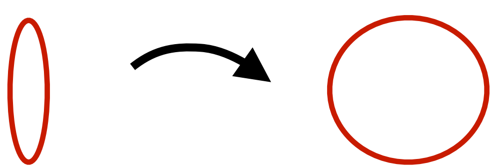
- Winding number: $W = \frac{1}{4\pi \mathrm{i}} \int_{-\pi}^{\pi} dk \, \mathrm{tr}(\Sigma Q_k^{-1} \partial_k Q_k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \vec{a} \cdot \left(\hat{\vec{n}}_k \times \partial_k \hat{\vec{n}}_k \right)$
- pure states: $\forall k : \; |\vec{n}_k| = 1$

$$\hat{\vec{n}}_k = \frac{\vec{n}_k}{|\vec{n}_k|}$$

- defined if topology of circle is preserved

$$\forall k : \; |\vec{n}_k| > 0$$

i.e. mixed states with “purity gap”



- circle collapses to line:

$$\exists k_0 : \; |\vec{n}_{k_0}| = 0$$

modes k_0 completely mixed

“purity gap” closes

- as long as the purity gap is finite, smoothly deform to a pure state

rationalization: J. C. Budich, S. Diehl, PRB (2015)

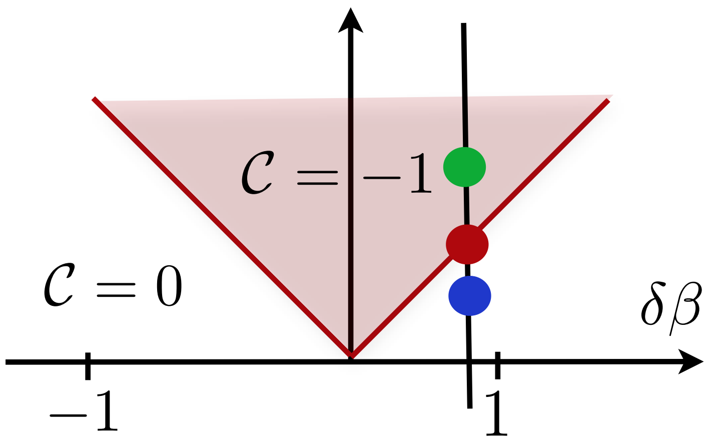
$$\vec{n}_k \to \hat{\vec{n}}_k \quad \text{for} \quad |\vec{n}_k| > 0 \quad \text{finite purity gap}$$

- in this case, topological invariant well defined

➡ two gaps required for topological stability: damping and purity gap

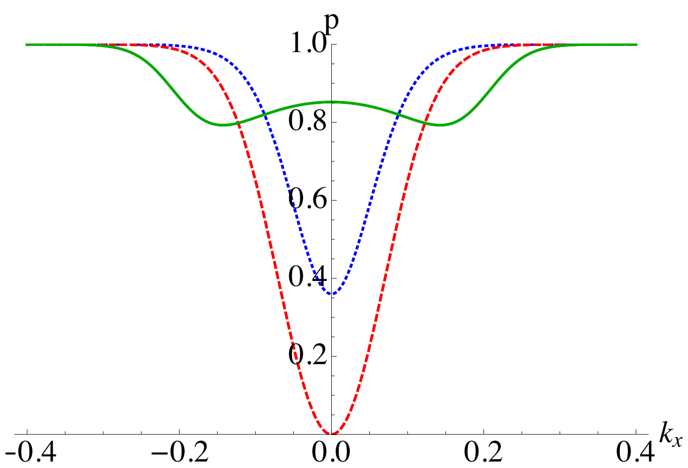
Nature of the Dissipative Topological Phase Transition

- phase diagram

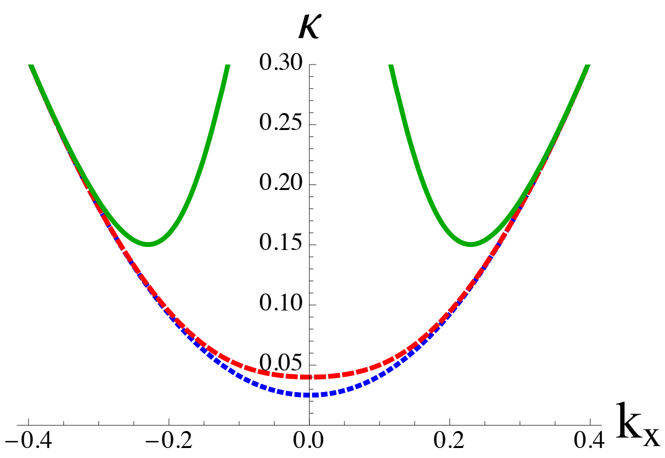


deviation from critical point

purity spectrum



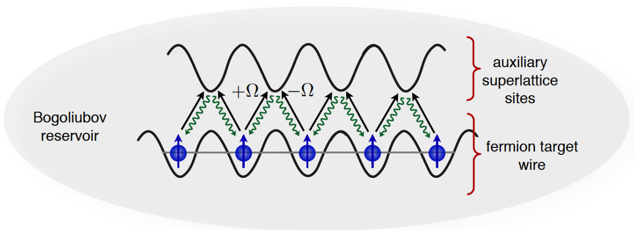
damping spectrum



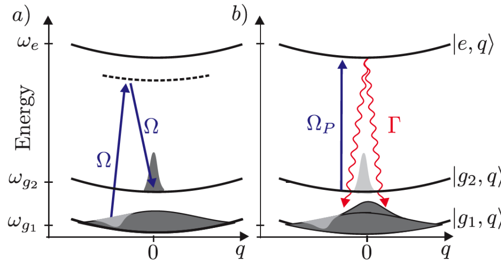
➡ topological phase transition by purity gap closing (non-critical)

Microscopic Model

- combine critical Lindblad operators with momentum selective pumping



SD, E. Rico, M. Baranov, P. Zoller, Nat. Phys. (2011); C. Bardyn et al. NJP (2013)



A. Griessner et al., NJP (2007)

$$\ell_i^C = C_i^{C\,\dagger} A_i^C$$

$$C_i^{C\,\dagger} = \sum_j v_{j-i}^C \psi_j^\dagger, \quad A_i^C = \sum_j u_{j-i}^C \psi_j$$

quasi-local near critical p-wave operators

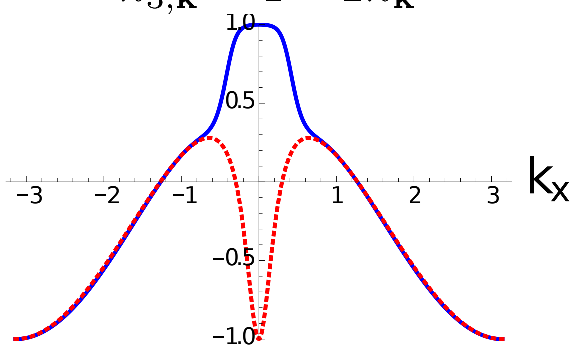
$$\tilde{\ell}_k^A = \sum_q \tilde{C}_{q-k}^{A\,\dagger} \tilde{A}_q^A$$

$$\tilde{C}_k^{A\,\dagger} = g_v \sum_i e^{-(k-\pi_i)^2/d_v^2} a_k^\dagger \qquad \tilde{A}_k^A = g_u \sum_i e^{-k^2/d_u^2} a_k$$

de-populating the low momentum modes

- self-consistent mean field theory for weak perturbation from exactly known pair state

$$n_{3,\mathbf{k}} = 1 - 2n_{\mathbf{k}}$$

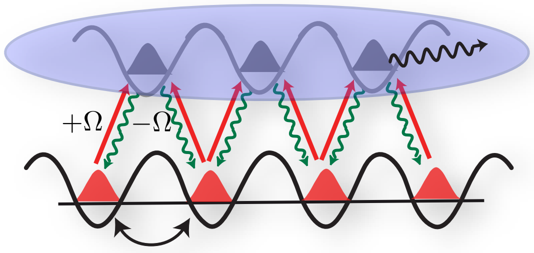


➡ full qualitative agreement with general analysis of quadratic master equation

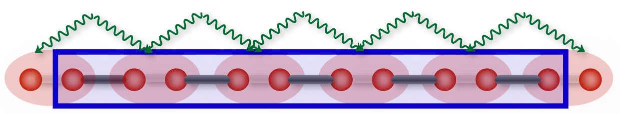
Summary: Topology by Dissipation

Tailored dissipation opens new perspectives for many-body physics with cold atom systems

- Targeting cooling of conventionally and topologically ordered quantum states



- 1D dissipative Kitaev chain: parallels Hamiltonian case



- 2D dissipative Chern insulator/superfluid: Harness intrinsic open system properties:
  - Competition of Topology and Locality in 2D
  - Critical Chern states require fine tuning
  - Stabilization of critical point into extended phase via hole plugging mechanism